1 Fig. 10 shows the speed of a car, in metres per second, during one minute, measured at 10 -second intervals.


Fig. 10
The measured speeds are shown below.

| Time $(t$ seconds $)$ | 0 | 10 | 20 | 30 | 40 | 50 | 60 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Speed $\left(v \mathrm{~m} \mathrm{~s}^{1}\right)$ | 28 | 19 | 14 | 11 | 12 | 16 | 22 |

(i) Use the trapezium rule with 6 strips to find an estimate of the area of the region bounded by the curve, the line $t=60$ and the axes. [This area represents the distance travelled by the car.]
(ii) Explain why your calculation in part (i) gives an overestimate for this area. Use appropriate rectangles to calculate an underestimate for this area.

The speed of the car may be modelled by $v=28-t+0.015 t^{2}$.
(iii) Show that the difference between the value given by the model when $t=10$ and the measured value is less than $3 \%$ of the measured value.
(iv) According to this model, the distance travelled by the car is

$$
\int_{0}^{60}\left(28 \quad t+0.015 t^{2}\right) \mathrm{d} t .
$$

Find this distance.

2 At a place where a river is 7.5 m wide, its depth is measured every 1.5 m across the river. The table shows the results.

| Distance across river $(\mathrm{m})$ | 0 | 1.5 | 3 | 4.5 | 6 | 7.5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Depth of river $(\mathrm{m})$ | 0.6 | 2.3 | 3.1 | 2.8 | 1.8 | 0.7 |

Use the trapezium rule with 5 strips to estimate the area of cross-section of the river.

3 Fig. 11 shows the cross-section of a school hall, with measurements of the height in metres taken at 1.5 m intervals from O .


Fig. 11
(i) Use the trapezium rule with 8 strips to calculate an estimate of the area of the cross-section.
(ii) Use 8 rectangles to calculate a lower bound for the area of the cross-section.

The curve of the roof may be modelled by $y=-0.013 x^{3}+0.16 x^{2}-0.082 x+2.4$, where $x$ metres is the horizontal distance from O across the hall, and $y$ metres is the height.
(iii) Use integration to find the area of the cross-section according to this model.
(iv) Comment on the accuracy of this model for the height of the hall when $x=7.5$.

4 Fig. 2 shows the coordinates at certain points on a curve.


Fig. 2

Use the trapezium rule with 6 strips to calculate an estimate of the area of the region bounded by this curve and the axes.

5 Fig. 10 shows a sketch of the graph of $y=7 x-x^{2}-6$.


Fig. 10
(i) Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$ and hence find the equation of the tangent to the curve at the point on the curve where $x=2$.

Show that this tangent crosses the $x$-axis where $x=\frac{2}{3}$.
(ii) Show that the curve crosses the $x$-axis where $x=1$ and find the $x$-coordinate of the other point of intersection of the curve with the $x$-axis.
(iii) Find $\int_{1}^{2}\left(7 x-x^{2}-6\right) \mathrm{d} x$.

Hence find the area of the region bounded by the curve, the tangent and the $x$-axis, shown shaded on Fig. 10.

